

Prop. Designer, It's on-design methods.

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Abstract

This paper presents the formulas and methods used to generate an on-design propeller of arbitrary design used by Prop. Designer. It also covers some of its implementation problems and limitations. This covers only part of what Prop. Designer does.

Nomenclature

a	= Axial interference factor	α	= Angle of attack
a'	= Rotational interference factor	β	= Blade angle of twist from plane of rotation
B	= Number of blades	ζ	= Displacement velocity
C_d	= Coefficient of drag	η	= Propeller efficiency
C_l	= Coefficient of lift	λ	= Speed ratio $V/\Omega R$
c	= Local propeller chord	ξ	= Non-dimensional radius
D	= Propeller diameter	ρ	= Fluid (air) density
d	= Hub diameter	σ	= Propeller solidity
J	= Advance ratio	ϕ	= Flow angle
j'	= Alternative advance ratio	ϕ_t	= Flow angle at tip
n	= Revs/Sec = Revs/minute/60	Ω	= Propeller angular velocity
v	= Air speed		

Introduction

Prop. Designer uses blade element theory to calculate the design and performance of a propeller blade at its design point. This is not the only theory for propeller design but it is one of the simplest and is suitable for human powered aeroplanes as the blade loading is so low.

As this is a computer program there is no need for small angle approximations to ease the calculations and Prop. Designer uses formulas derived without using these approximations. In practice this makes a small difference under normal circumstances but it can make a difference in the off-design situation which is not covered in this paper.

As Prop Designer is written in Visual Basic 6, an over 10 year old language that is unsupported by Microsoft, no code is presented and various subroutines are only mentioned in outline.

Basic Theory

The propeller blade is divided up into sections and the thrust and torque is calculated for each section and then numerically integrated to get the total thrust, torque and power for the blade. Multiplying by the number of blades gives the totals for the whole propeller. From this the propeller's efficiency can be calculated.

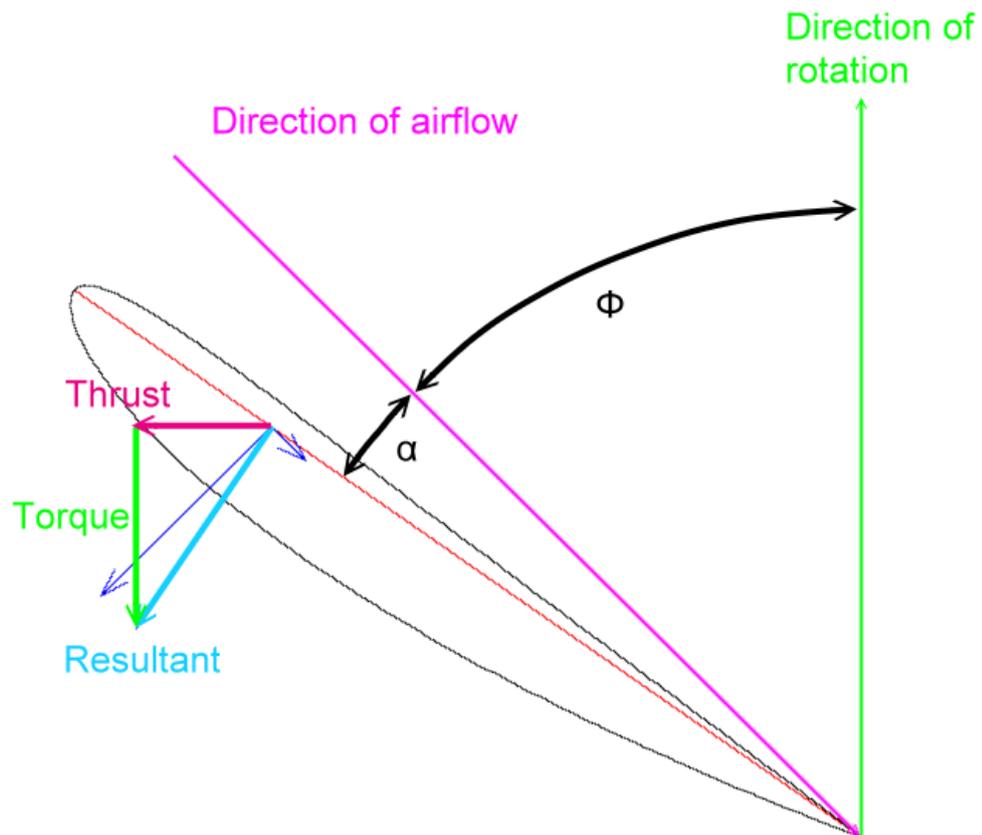


Diagram of basic forces on a blade element

The Coefficient of Power

This is just a measure of the amount of power generated for the swept area of the propeller. Prop. Designer uses a slight modification of the standard definition in that it subtracts the area of the hub from the total area. This normally only increases the coefficient of power by a few percent.

$$C_p = Q \rho v^2 (D^5 - d^5) \tag{1}$$

And this would normally have a value of between 0 and 1.5 for reasonable results, preferably between 0 and 1.

Initial calculations

To set up the major loop to step from the hub to the tip, Prop. Designer initializes an array of size equal to (the length of a blade from hub to tip in millimetres) + 1 that will hold values for thrust, torque, power, chord, blade angle and local efficiency. The array holds other values than those mentioned because it also holds values for the off-design point as well as values that can be later graphed. These are Reynolds number and angle of attack.

Angular velocity

$$\Omega = 2 \pi n \quad (2)$$

Advance ratio

$$J = \frac{v}{n d} \quad (3)$$

An alternative advance ratio you may come across. This is for the screen display only.

$$j' = \frac{v}{\Omega D/2} \quad (4)$$

Main loop

The main loop is executed once for each point along the blade from the hub to the tip. There was no particular reason to do it that way round except measurements are made from the hub but it might be better to step from the tip to the hub.

Local propeller solidity

$$\sigma = \frac{B c}{2 \pi r} \quad (5)$$

(5) is the blades' solidity and should not exceed 1.

Non dimensional radius

$$\xi = r/R \quad (6)$$

Initial estimate of angle. This will be refined later.

$$\phi = \tan^{-1}\left(\frac{v}{\Omega r}\right) \quad (7)$$

This value is initially also saved as "old estimate", the reason will become clear shortly.

Initial estimate of Reynolds number. In practice the properties of the aerofoil are found once for this estimate as they do not vary greatly.

$$Re = \sqrt{v^2 + (n \pi 2 r^2)} c \frac{\rho}{\mu} \quad (8)$$

C is determined from a table of chord values at different radii and linear interpolation. The minimum number of values in the table is 2, the hub chord and the tip chord. Prop Designer limits the minimum value for chord to 10 mm.

C_l is determined from a graph of C_l versus position along blade. In practice it uses the Bezier curve the graph is drawn with and a Binary chop search to get the C_l accurate to about a 10^{th} of a millimetre.

C_d is determined from a database of values and linear interpolation.

α is determined from a database of values and linear interpolation.

Inner loop

A new value for the angle ϕ is generated from the previous value of ϕ and the new value of ϕ just calculated by adding the 2 value together and dividing by 2. This then becomes the next “previous value of ϕ ”. This enables a convergence in a few iterations of the loop

If the propeller is open and not shrouded then a value for the Prandtl tip loss should be calculated.

$$F = \left(\frac{2}{\pi}\right) \cos^{-1}(e^{-f}) \quad (9)$$

Where

$$f = \frac{(B/2)(1-\xi)}{\sin(\phi_t)} \quad (10)$$

Where

$$\tan \phi_t = \lambda (1 + \zeta / 2) \quad (11)$$

For a shrouded or ducted propeller the value is always 1 and so there is no need to calculate it.

The forces of thrust are

$$C_y = C_l \cos \phi - C_d \sin \phi \quad (12a)$$

The forces of torque are

$$C_x = C_l \sin \phi + C_d \cos \phi \quad (12b)$$

The interference factors are

$$a = \frac{\sigma K}{(F - \sigma K)} \quad (13a)$$

$$a' = \frac{\sigma k'}{F + \sigma k'} \quad (13b)$$

Where

$$K = \frac{C_y}{(4 \sin^2 \phi)} \quad (14a)$$

$$K' = \frac{C_x}{(4 \cos \phi \sin \phi)} \quad (14b)$$

$$\tan \phi = \left[\frac{v(1+a)}{\Omega r (1-a')} \right] \quad (15)$$

(16) is the new value for the angle of the airflow. If this is close to the previous value then you can exit the loop and save the values ready for the next section. The error should be below 0.1 of a degree, Prop Designer uses 0.0001 of a Radian, about 0.005 of a degree which is smaller than it needs to be.

The results need to be saved.

Note that this calculates ϕ , the local airflow angle, the blade angle, β is $\phi + \alpha$.

Back to the main loop

The speed of the air over the propeller can be calculated as well as the actual Reynolds number.

The local thrust is

$$C'_t = \frac{\left(\frac{\pi^3}{4}\right) \sigma C_y \xi^3 F^2}{[(F + \sigma K') \cos \phi]^2 R} \quad (16)$$

The local torque is

$$C'_p = \frac{C'_t \pi \xi C_x}{C_y} \quad (17)$$

Local efficiency is

$$\eta' = \frac{C'_t J}{C'_p} \quad (18)$$

Integration

Prop Designer integrates the torque and thrust results using Simpson's rule although it could probably get away with using the trapezium rule if the propeller blade is over 1/2 metre in length.

While I have not investigated in any great detail for the minimum number of data points it needs for a precise answer, it is greater than 10 and less than 100 as the results for 100 points are very close to the results for 1000 points. The reason for needing more than 10 points is that Simpson's rule is not a good approximation with the low number of points as the thrust varies too greatly between points for it to approximate accurately.

Results

Torque and thrust are integrated and calculated and the overall efficiency is then calculated from the result rather than integrating the results.

Accuracy

How do you know the results are valid? Well you have to use some test designs to see if your results are close to the test cases. Any serious discrepancy has to be investigated. Prop Designer was compared not just to previous versions of itself but to Xrotor and to known propeller designs. This will show up any obvious bugs, the results are presented to an accuracy that are practical

Limitations

Blade element theory is limited to low to medium blade loadings, Prop. Designer does not have any corrections for compressibility and so blade tip speeds are limited to about half the speed of sound, about 150 metres/second at sea level. As www.propdesigner.co.uk covers human and low powered aircraft this is not considered a problem.

The maths is only stable when the lift coefficient is increasing with angle of attack so this can cause problems near the stall. It will not cope with zero forward speed and checks have to be added for zero C_l , zero blade length and zero chord which all generate division by zero errors.

Any program like Prop Designer can be pushed beyond it's design limits at which point the results become increasingly unreliable.

Conclusion

Using the formulas, methods and allowing for various problems presented here, it is possible to write a program to check the design of arbitrary propellers quickly and accurately.

References

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